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EFFECT OF FRACTURE RATE ON THE DYNAMICS OF THE INTERACTION OF AN IMPACT LOAD PULSE WITH THE SURFACE OF A SOLID

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Studies of cleavage phenomena during the reflection of shock waves from the free surface of a body [1, 2] provide unique information about the strength properties of materials in the submicrosecond range. Under these conditions, however, the time to fracture is comparable with the loading time and as a result the experimental values of the cleavage strength of material, which is not a comprehensive characteristic, are not unique; it is thus necessary to speak of the breaking strength as a function of the strain rate as well as other state parameters. A number of papers (see, e.g., [3, 4]) have developed a semi-empirical continuum-kinetic model of fracture, which gives an acceptable description of particular cases when used in problems of mathematical simulation of shock-wave phenomena. At the same time, information must be obtained about the kinetic fracture laws directly from analysis of experimental data. Such information in implicit form is contained by the velocity profiles of the surface of the test specimen [5]. The fracture of the material after reflection of a shock wave from the free surface of a body and the attendant relaxation of tensile stresses give rise to a compression wave, a so-called cleavage pulse. Clearly, in the case of instantaneous fracture the cleavage pulse should have the steepest leading edge and the largest amplitude. It is intuitively clear that a longer time to fracture reduces the slope of the cleavage pulse. A prolonged decrease in velocity against the background of its damped oscillations has been also observed in experiments.

Our aim was to analyze wave processes in a fracturing medium upon reflection of a compression pulse from the free surface and to study the possibility of obtaining data on the fracture rate directly from measurements of the velocity profiles of the surface of the specimen.

Formulation and Solution of the Problem. In the acoustic approximation we consider the evolution of a triangular compression pulse after its reflection from the free surface of a specimen, which develops at negative pressure. We assume that fracture begins when the tensile stresses reach the critical value P_c and is characterized by a specific pore volume v_p. The total specific volume of the medium is equal to the sum of v_p and the specific volume of the solid component v_s: $v = v_p + v_s$. We use the simplest fracture kinetics: the rate of change of v_p depends linearly on the pressure P and is zero if $P > 0$ and $v_p = 0$. The system of hydrodynamic equations, closed by the kinetic equation and the equation of state, has the form (in Lagrange's variables)

$$\frac{\partial v}{\partial t} - \frac{1}{\rho_0}\frac{\partial u}{\partial h} = 0,\ \frac{\partial u}{\partial t} + \frac{1}{\rho_0}\frac{\partial P}{\partial h} = 0,\ \frac{\partial v_p}{\partial t} + \frac{P}{\rho_0^2 c_0^2 \tau_\mu} = 0,\ P = \rho_0^2 c_0^2 (1/\rho_0 - v + v_p), \tag{1}$$

where t is the time; h is the Lagrange coordinate; u is the mass velocity; ρ_0 and c_0 are the initial density and the velocity of sound; and τ_μ is the characteristic relaxation time of the fracture process, corresponding to the bulk viscosity $\mu = \rho_0 c_0^2 \tau_\mu$. In the equation of state the pressure is determined from the solid component $v_s = v - v_p$.

Figure 1 shows the flow pattern in the t–h plane. In region 1 the incident wave and the reflected wave do not interact and the dependence of the mass velocity and pressure on the coordinates and time corresponds to a triangular compression pulse:

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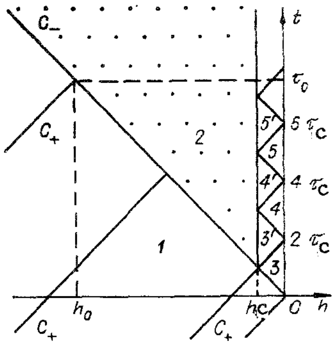

Fig. 1

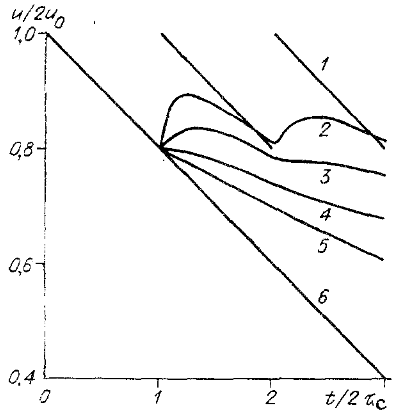

Fig. 2

$$u(h,\ t)=u_0-k(c_0t-h),\ P(h,\ t)\ =\rho_0c_0u(h,\ t). \tag{2}$$

Here u_0 is the maximum mass velocity; and k is a constant characterizing the pulse length $2h_0$:

$$h_0=-c_0\tau_0=-u_0/(2k).$$

In region 3 the incident wave and the wave reflected from the free surface h = 0 interact, causing tensile stresses. The absolute value of these stresses is below the critical value, whereby the medium does not fracture and the solution satisfying the free-surface condition is written as

$$u(h,\ t)=2(u_0-kc_0t),\ P(h,\ t)=2\rho_0c_0kh. \tag{3}$$

At $h = h_c$ and $\tau = \tau_c = -h_c/c_0$ the pressure reaches the threshold P_c and the material fractures in region 2. The flow here is determined by the solution of system (1) with boundary conditions at $h = h_c$ and $h \to -\infty$ and initial conditions on the C_- characteristic, on which the functions (apart from v_p) undergo a jump. Because of the stress relaxation during fracture the pressure along the C_- characteristic may (for certain values of τ_c) be above P_c and the fracture region has a more complicated zone structure than in Fig. 1. To simplify the calculations, therefore, we assume that once the fracture threshold has been reached in section h_c the medium "becomes weaker" at lower values of h. From the solution obtained below we shall see under what conditions and how the fracture threshold should change at $h \le h_c$ for the fracture region to have the shape shown in Fig. 1.

Let us find the solution in region 2. For this purpose we eliminate v_p and v from (1) and change the independent variables: $T = t + h/c_0$, $x = h$. The fracture zone is mapped onto part of the fourth quadrant of the T–x plane: $T \ge 0$, $x \le x_c$. The Laplace transform in T takes the resulting system of two equations in partial derivatives into a system of ordinary differential equations.

$$\begin{aligned}&\frac{d\hat{u}}{dx}+\frac{s}{c_0}\hat{u}+\left(s+\frac{1}{\tau_\mu}\right)\frac{\hat{P}}{\rho_0c_0^2}=\frac{1}{\rho_0c_0^2}(P(x,0)+\rho_0c_0u(x,0)),\\&\frac{d\hat{P}}{dx}+\frac{s}{c_0}\hat{P}+\rho_0s\hat{u}=\frac{1}{c_0}(P(x,0)+\rho_0c_0u(x,0))\end{aligned} \tag{4}$$

(s is the Laplace variable and $\hat{u}$ and $\hat{P}$ are the Laplace transforms of the mass velocity and pressure). The initial values of u and P as $T \to +0$, which enter as a combination that is a Riemann J_+-invariant, are transferred to the right side of (4) [6]. It is not necessary, therefore, to determine u and P separately to the right of the jump on the C_- characteristic: they are found directly from the solution of the system. The value of the invariant is found from the continuity condition at the jump on the basis of its value in region 1. By (2) we obtain

$$P(x,0)+\rho_0 c_0 u(x,0)=2\rho_0 c_0(u_0+2kx)\,\theta(x-x_0),$$

where $\theta(x)$ is the Heaviside unit function; and $x_0 = h_0$.

The general solution in the fracture region, which is bounded as $x \to -\infty$, has the form

$$\widehat{P}(x,s)=\frac{2k\rho_0 c_0}{s}\Big[-2c_0\tau_\mu+\frac{s}{\Delta}\Big(\frac{1}{\lambda_1}\exp(\lambda_1(x-x_0))-$$
$$-\frac{1}{\lambda_2}\exp(\lambda_2(x-x_0))\Big)\Big]\theta(x-x_0)+a\exp(\lambda_1 x),$$
$$\widehat{u}(x,s)=\frac{2k}{s}\Big[2(x-x_0+c_0\tau_\mu)-\frac{1}{\lambda_1}\exp(\lambda_1(x-x_0))-\frac{1}{\lambda_2}\exp(\lambda_2(x-x_0))\Big]\times \tag{5}$$
$$\times\,\theta(x-x_0)-\frac{\Delta}{\rho_0 c_0 s}a\exp(\lambda_1 x),$$
$$\lambda_{1,2}=-\frac{s}{c_0}\pm\frac{\Delta}{c_0},\quad \Delta=\sqrt{s(s+1/\tau_\mu)}.$$

Constant a is found from the continuity condition of the Riemann J_--invariant at $x = x_c$. The complication is that in regions 3', 4', etc. (see Fig. 1) the functional dependence of J_- on the coordinates and the time is different and in each subsequent region the invariant is determined only after the solution has been found in the previous region. Let us find the value of a in the interval $0 \leq T \leq 2\tau_\mu$. In region 3, by (3) we obtain

$$J_-=-2\rho_0 c_0[u_0-k(c_0 t+h)]=-2\rho_0 c_0(u_0-kc_0 T). \tag{6}$$

Since the J_--invariant persists along the C_- characteristics, Eq. (6) gives its value in region 3'. Applying the Laplace transformation to (6) and setting the resulting expression equal to the $\hat{J}_-$-invariant in the fracture region, which follows from (5) for $x = x_c$, we find a:

$$a=-\frac{2k\rho_0}{\lambda_2}\Big[2x_c+4c_0\tau_\mu+\frac{c_0}{s}+\frac{c_0}{\Delta}\Big(\frac{\lambda_2}{\lambda_1}\exp(\lambda_1(x_c-x_0))-$$
$$-\frac{\lambda_1}{\lambda_2}\exp(\lambda_2(x_c-x_0))\Big)\Big]\exp(-\lambda_1 x_c). \tag{7}$$

Equations (5) and (7) determine the solution in the fracture region for $0 < T \leq 2\tau_c$ in Laplace transforms. Some results can be obtained without going to the originals. For instance, using the familiar property of the Laplace transformation [7] $\lim\limits_{s\to\infty} sF(s)=F(0)$, we find the value of the pressure to the right of the jump along the C_- characteristic

$$P=-4k\rho_0 c_0^2\tau_\mu\Big[\Big(1-\exp\Big(\frac{h-h_0}{2c_0\tau_\mu}\Big)\Big)\theta(h-h_0)+\exp\Big(\frac{h-h_0}{2c_0\tau_\mu}\Big)-$$
$$-\Big(1-\frac{\tau_c}{2\tau_\mu}\Big)\exp\Big(\frac{h-h_c}{2c_0\tau_\mu}\Big)\Big]. \tag{8}$$

From (8) it follows that when fracture has begun at point h_c, τ_c the pressure along the C_- characteristic continues to decrease if $\tau_\mu < \tau_c/2$ and conversely, begins to increase if $\tau_\mu > \tau_c/2$ tending to $-4k\rho_0 c_0^2\tau_\mu$ in both cases (we consider the most interesting case, $h \geq h_0$). The pressure remains constant at P_c when $\tau_\mu = \tau_c/2$. The above assumption about the "weakening" of the material is important at a low fracture toughness, when $\tau_\mu < \tau_c/2$ and the solution obtained remains valid if we assume that the fracture threshold at $h < h_c$ drops to $4k\rho_0 c_0^2\tau_\mu$ in absolute value.

Let us find the free surface at $2\tau_c \leq 4\tau_c$. To do this we exploit the circumstance that the Riemann J_+-invariant persists along the C_+ characteristics. We find is value on the free surface is $\rho_0 c_0 u(0, t)$ and at $h = h_c$ from the solution obtained in the fracture region:

$$\frac{\widehat{J}_+(x_c,s)}{\rho_0 c_0}=\frac{4k}{s}(x_c-x_0)+\frac{4k\lambda_1}{s\lambda_2}(x_c+2c_0\tau_\mu)+\frac{2kc_0\lambda_1}{s^2\lambda_2}+\frac{8k}{c_0\lambda_2^2}\exp(\lambda_2(x_c-x_0)).$$

Using the familiar inversion formulas and the properties of the Laplace transformation [7, 8], we obtain an expression for the velocity of the free surface at $2\tau_c \leq t \leq 4\tau_c$:

$$
\begin{gathered}
\frac{u(0,t)}{2u_0} = 1 - \frac{1}{\delta_0}\left[1 + (1-2\delta_\mu)F_1(z) - \delta_\mu F_2(z) - 2\delta_\mu F_3(z)\right], \\
F_1(z) = \exp(-z)\,[I_0(z) + I_1(z)] - 1, \\
F_2(z) = \exp(-z)\,[2z(I_0(z) + I_1(z)) + I_0(z)] - z - 1, \\
F_3(z) = \exp(-z + z_c)\int\limits_{z_c}^{z-z_c}\left[I_0(z - z_c - \xi) + \left(1 - \frac{1}{z - z_c - \xi}\right)I_1(z - z_c - \xi)\right]\times \\
\times I_0\left(\sqrt{\xi^2 - z_c^2}\right)d\xi\;\theta(z - 2z_c), \\
z = \frac{t - 2\tau_c}{2\tau_\mu},\; z_c = \frac{x_c - x_0}{2c_0\tau_\mu} = \frac{\tau_0 - \tau_c}{2\tau_\mu},\; \delta_0 = \frac{\tau_0}{\tau_c},\; \delta_\mu = \frac{\tau_\mu}{\tau_c}
\end{gathered} \tag{9}
$$

(I_0 and I_1 are modified Bessel functions of order 0 and 1). We note that function F_3 is nonzero only when the load pulse length is less than $4\tau_c$. Usually, for low-strength media, this condition is not satisfied and then Eq. (9) simplifies considerably since $u(0, t)$ is expressed explicitly in terms of modified Bessel functions.

Having determined the velocity of the free surface in region 4 (see Fig. 1), we can find the value of the J_--invariant and, repeating the previous steps, find the solution in the fracture region at $2\tau_c \leq T \leq 4\tau_c$. Omitting the intermediate manipulations, we give the expression for the velocity of the free surface in the interval $2\tau_c \leq t \leq 6\tau_c$:

$$
\begin{gathered}
\frac{u(0,t)}{2u_0} = 1 - \frac{1}{\delta_0}\Big\{1 + (1-2\delta_\mu)F_1(z) - \delta_\mu F_2(z) + \Big[\delta_\mu F_2\left(z - \frac{1}{\delta_\mu}\right) - \\
- (1-2\delta_\mu)\,\Phi_1\left(z - \frac{1}{\delta_\mu}\right) + \delta_\mu\Phi_2\left(z - \frac{1}{\delta_\mu}\right)\Big]\theta\left(z - \frac{1}{\delta_\mu}\right)\Big\}, \\
\Phi_1(z) = 1 - 2\exp(-z)\,[I_0(z) + I_1(z) - I_1(z)/z], \\
\Phi_2(z) = z + 4 - 4z\exp(-z)\,[I_0(z) + I_1(z)] - 2\exp(-z)\times \\
\times [2I_0(z) + I_1(z)].
\end{gathered} \tag{10}
$$

In (10) the pulse length $2\tau_0$ is assumed to be greater than $6\tau_c$. Otherwise F_3 and the function Φ_3 of similar structure. The construction of the solution can be continued for longer times. With each step, however, this becomes more cumbersome. Moreover, it is the first several oscillations of the velocity, described by Eq. (10), that are of interest in practice.

Analysis of the Solution. Let us examine the dependence of the velocity of the free surface on the relaxation time of the fracture process at a finite P_c. First we consider the limiting cases. As $\tau_\mu \to \infty$ the arguments of F_i and Φ_i tend to zero. Expanding the modified functions for small z [8], we obtain $u(0, t) = 2(u_0 - kc_0t)$ for $2\tau_c \leq t \leq 6\tau_c$, which accords with the solution (2) and corresponds to the absence of fracture. In the second limiting case ($\tau_\mu \to 0$) we have

$$
\begin{gathered}
u(0,t) = 2[u_0 - kc_0(t - 2\tau_c)] \\
\text{for } 2\tau_c \leq t \leq 4\tau_c, \\
u(0,t) = 2[u_0 - kc_0(t - 4\tau_c)] \\
\text{for } 4\tau_c \leq t \leq 6\tau_c,
\end{gathered}
$$

as should be expected for media that fracture without resistance after the tensile stresses reach the critical value.

Figure 2 showed the velocity profiles plotted from Eq. (10) for $\delta_\mu = 0$; 0.05, 0.2, 0.5, 1, and ∞ (lines 1-6) and $\delta_0 = 5$. Tabulated values of the Bessel functions were taken from the handbook [8]. Since the velocity is inversely proportional to the δ_0, a change in the pulse length reduces to just a change in scale along the abscissa axis and in this sense the graph in Fig. 2 is universal. The most characteristic feature of the solution is a critical value δ_μ^*, which separates two different flow regimes: at $\delta_\mu < \delta_\mu^*$ the velocity of the free surface oscillate, i.e., fracture manifests itself in a cleavage pulse at the profile u(0, t); at $\delta_\mu > \delta_\mu^*$ the monotonic decrease in velocity continuous after the onset of fracture. Let us determine the critical value of the fracture toughness, for which purpose we find derivative of the velocity with respect to time for $2\tau_c < t \le 4\tau_c$:

$$\frac{d}{dt}\left(\frac{u(0,t)}{2u_0}\right) = \frac{1}{2\tau_0}\left[\left(\frac{1}{\delta_\mu} - 2\right)\exp(-z)\frac{I_1(z)}{z} + F_1(z)\right]. \tag{11}$$

In particular, at $t = 2\tau_c + 0$ (the derivative has a jump at $t = 2\tau_c$)

$$\frac{d}{dt}\left(\frac{u(0,2\tau_c)}{2u_0}\right) = \frac{1}{4\tau_0}\left(\frac{1}{\delta_\mu} - 2\right). \tag{12}$$

The critical value of δ_μ^*, determined from the condition for the derivative to become zero at $t = 2\tau_c + 0$, is 0.5. Equation (12) is sufficiently general, does not depend on the specific model of fracture and is determined only by the initial pore growth rate, which we denote by $\dot{v}_p$. It follows from (1) that $\dot{v}_p = 2k/(\rho_0\delta_\mu)$ at $P = P_c$. Also introducing the growth rate $\dot{v}$ of the specific volume in the unloaded part of the decreasing pulse ($\dot{v}$ is constant and equal to k/ρ_0), we recast Eq. (12) in the form

$$\frac{d}{dt}\left(\frac{u(0,2\tau_c)}{2u_0}\right) = \frac{1}{8\tau_0}\left(\frac{\dot{v}_p}{\dot{v}} - 4\right). \tag{13}$$

This suggests that a cleavage pulse at the velocity profile of the free surface is observed only when the maximum pore growth rate is more than four times the growth rate of the specific volume in the decreasing load pulse. Formula (13) also provides a way of finding the initial fracture rate from the slope of the leading edge of the cleavage pulse.

We determine the position of t_m and the amplitude of the first velocity maximum u_m, equating the derivative (11) to zero. At a fracture toughness close to the critical value ($1 - \delta_\mu/\delta_\mu^* \ll 1$), we obtain

$$t_m/2\tau_c \simeq 2 - \delta_\mu/\delta_\mu^*, \quad u_m/2u_0 \simeq 1 - (1/\delta_0)\left[1 - (1 - \delta_\mu/\delta_\mu^*)^2/2\right].$$

As $\delta_\mu \to 0$

$$t_m/2\tau_p \simeq 1 + (\delta_\mu/2\pi)^{1/3}, \quad u_m/2u_0 \simeq 1 - (3/\delta_0)(\delta_\mu/2\pi)^{1/3},$$

i.e., as δ_μ decreases from δ_μ^* to 0 the velocity maximum increases monotonically, first shifting to the right, and then again approaching $t = 2\tau_c$. The maximum $t_m/2\tau_c \simeq 1.31$ is reached at $\delta_c \simeq 0.18$ sec, which corresponds to the velocity maximum $u_m/2u_0 \simeq 1 - 0.80/\delta_0$. The dependence shown in Fig. 2 for $\delta_\mu = 0.2$ is close to this nature of the motion. In contrast to Eq. (13) all of the results pertaining to the location and amplitude of the velocity maximum are intimately bound up with the specific model of fracture and information about the shape of the cleavage pulse can be used to determine the characteristic relaxation time in Eq. (1).

Interesting results also follow from analysis of the velocity profile of the secondary circulation in the cleavage plate: $4\tau_c \le t \le 6\tau_c$. First of all, it is easily ascertained that the velocity given by Eq. (10), which is continuous along with its first derivative at $t = 4\tau_c$, and the second minimum when it exists, is observed at t is slightly larger than $4\tau_c$: $t/4\tau_c \simeq 1 + 2\delta_\mu^2$, $\delta_\mu \to 0$. We also see from Fig. 2 that there is a second critical value δ_μ^{**}, which is characterized by the fact that a cleavage pulse is observed in the interval $\delta_\mu^* > \delta_\mu > \delta_\mu^{**}$, but further on the velocity decreases without characteristic minima and maxima: $\delta_\mu^{**} \approx 0.13$.

Thus far we have studied the velocity of the free surface. Information about the pressure distribution in the fracture region is important for analysis of cleavage phenomena. For simplicity we study only part of this region ($0 < T \le 2\tau_c$), where the solution is given by Eqs. (5) and (7). We shall show that at small δ_μ the pressure is virtually constant in the fracture region, for which we find the partial derivative of P with respect to T. This is done most simply at T = +0, by applying the Laplace transform, which was used in the derivation of formula (8). We can easily show that in this case

$$\frac{\partial P}{\partial T} = \frac{k\rho_0 c_0^2}{2\delta_\mu} \exp\left(\frac{x - x_c}{2c_0\tau_\mu}\right),$$

i.e., the derivative is exponentially small. In the approximation under consideration, therefore, the pressure is constant and is equal to the value behind the jump, which is given by Eq. (8):

$$P \simeq -4k\rho_0 c_0^2 \tau_\mu. \tag{14}$$

Near the cleavage plane the nature of the pressure variation is different. Let us examine the details of P at $h = h_c - 0$. It is easy to invert the Laplace transform and obtain an explicit expression for the pressure

$$P(h_c,\ t) = P_c(1 - \delta_\mu y + (1 - 2\delta_\mu)F_1(y) - \delta_\mu F_2(y)),$$
$$\tau_c \leq t \leq 3\tau_c \text{ and } \tau_0 \geq 2\tau_c, \quad y = (t - \tau_c)/2\tau_\mu, \tag{15}$$

from which, e.g., it follows that the derivative of the pressure is positive in the time interval under study at $\delta_\mu \leq \delta_\mu^*$, i.e., the pressure in the cleavage plane increases with time, reaching a maximum at $t = 3\tau_c$. At $\delta_\mu \leq 0.11$ this maximum is positive and its amplitude at first grows with decreasing fracture toughness to 0.06 P_c at $\delta_\mu \simeq 0.02$ and then falls to zero. The time at which the pressure in the cleavage plane changes sign is $t \simeq \tau_c\left(2 + 1.5\sqrt{\pi\delta_\mu} + \frac{9\pi - 20}{8}\delta_\mu\right)$.

Let us consider the porosity distribution in the fracture region. In the approximation of small values of τ_μ, when the pressure is practically constant and is determined by (14), we obtain

$$v_p(h,\ t) \simeq (4k/\rho_0)(t + h/c_0), \tag{16}$$

i.e., the law of pore growth is independent of the relaxation time and can be found directly from analysis of flow in a medium that fractures without resistance. Deviations from (16) are observed only near the cleavage plane. At $h = h_c - 0$ (like the pressure, v_p jumps in the section h_c) the dependence of porosity on time follows directly from (15):

$$v_p(h_c,\ t) = (4k\tau_c/\rho_0)[y - \delta_\mu y^2/2 + (1 - 2\delta_\mu)F_2(y) - \delta_\mu F_4(y)],$$
$$F_4(y) = y\exp(-y)[I_0(y) + I_1(y)/3 + (4y/3)(I_0(y) + I_1(y))] - y^2/2 - y. \tag{17}$$

The maximum specific pore volume is reached near the cleavage plane and at $\tau_c \leq t \leq 3\tau_c$ can be found from (17).

In the general case solving for the pressure and specific pore volume is a rather laborious process. For the sake of illustrating the conclusions, therefore, we carried out a numerical simulation of the system of gas-dynamic equations (1) by the method of characteristics [9], splitting them according to the physical processes. The coordinate distributions of the pressure and the specific pore volume at $\delta_0 = 5$ and $\delta_\mu = 0.05$ are given in Fig. 3. The numbers indicate the time (in microseconds) from the arrival of the shock wave at the free surface (τ_c = 2/3 μsec) and the dashed lines represent the P and v_p curves, plotted from Eqs. (14) and (16) at t = 3 μsec. The approximations (14) and (16) rather accurately give the maximum tensile stresses in the fracture zone and indicate the linear nature of the growth of the specific pore volume as a function of time and coordinate. We also note that at t = 2 μsec the pressure at $h = h_c - 0$ is positive and agrees with calculation from Eq. (15).

The analysis has thus far been conducted on the assumption that the fracture threshold P_c is finite. Let us consider the limiting case $P_c = 0$, which corresponds to thresholdless fracture of a medium. The passage to the limit directly in (10) obviously does not solve the problem posed, since regions 3, 4, ... constrict to a point and their number in the fixed interval of time tends to infinity. At the same time, the general solution (5) remains valid and the constant a is determined from the condition $\hat{P} = 0$ at x = 0. The velocity and pres-

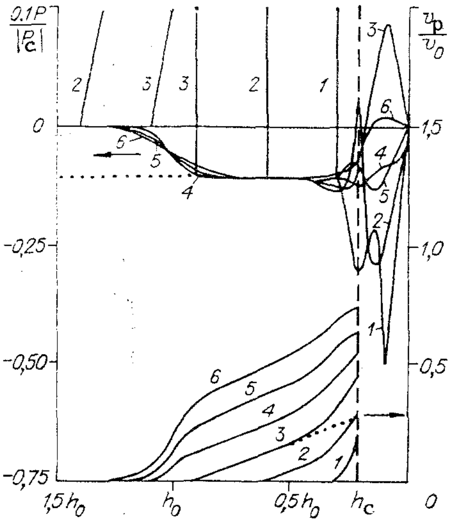

Fig.3

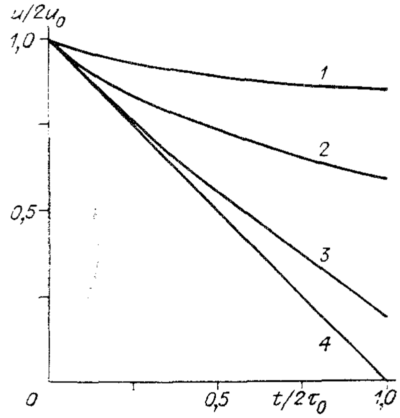

Fig. 4

sure fields are described by simpler expressions than at a finite P_c. In particular, the velocity of the free surface at $t \leq 2\tau_0$ has the form

$$\frac{u(0,\,t)}{2u_0} = 1 - \frac{\tau_\mu}{\tau_0}\left[F_2\left(\frac{t}{2\tau_\mu}\right) + \frac{t}{2\tau_\mu}\right]. \tag{18}$$

The velocity profiles at $\tau_\mu/\tau_0 = 0.01$; 0.1; 1; and ∞ (lines 1-4) in Fig. 4 were plotted from Eq. (18) in coordinates $u(0,\, t)/2u_0 - t/2\tau_0$. The velocity $u(0,\, t) \to 2u_0$ as $\tau_\mu \to 0$, as it should for media with no strength. In the second limiting case ($\tau_\mu \to \infty$) the velocity $u(0,\, t) = 2u_0 - 2kc_0t$, i.e., the specimen does not fracture. At $t \to 0$ the dependence $u(0,\, t)$ pertains to the velocity profile that would be observed in the absence of fracture. This makes it impossible in principle to determine the fracture threshold when it is small and no cleavage pulse is recorded on the experimental profile.

In conclusion, we note that all of the flow regimes obtained within the framework of the model are observed in practice. Indeed, the solution (10) for $\delta_\mu > \delta_\mu^{**}$ corresponds to the well-known nature of the cleavage fracture of many structural materials [2]. The case $\delta_\mu^* < \delta_\mu < \delta_\mu^*$, when only the first cleavage pulse is recorded distinctly, corresponds to the behavior of highly extended elastomers. For example. Weirick [10], who studied cleavage in simulants of solid propellant, whose free-surface velocity profiles showed only the first cleavage pulse with a subsequent monotonic drop. Kamykov et al. [11] were unable to record a cleavage pulse in experiments with rubber. A similar result was obtained independently in [12]. The model of thresholdless fracture (18) can be used to describe these experiments.

In summary, within the framework of acoustics an analytical expression has been derived for the velocity of a free surface during plastic fracture of a material. Critical values at which the flow regime changes have been found. A method not linked with any specific model of fracture is proposed for determining the initial growth rate of the specific pore volume from the slope of the leading edge of the cleavage pulse.

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ONE-DIMENSIONAL PROJECTION OF A LIQUID SHELL BY AN EXPLOSIVE CHARGE

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Introduction. The problem considered here is associated with the cavitational rupture of a liquid with a free surface under an explosive load. The concept of cavitational rupture is based on the fact that cleavages arise in cavitating liquids behind the leading edge of intense rarefaction waves in underwater explosions at shallow depths [1]. Detailed experimental analysis of the nature and dynamics of cavitation effects has shown that the rupture process has distinctive features and has a number of stages: a) unrestricted growth (a necessary condition) of cavitation nuclei up to the "bulk" bubble density, corresponding to a bulk concentration of 0.5-0.75; b) formation of foam-type structures and their breakup into fragments, i.e., cleavages; c) the transformation of the cavitating cleavages into a drop structure (structure of a splash dome on the free surface [1]).

Under an explosive load a real liquid, containing microinhomogeneities as cavitation nuclei being in essence a two-phase medium for rarefraction waves, is transformed into a gas-drop state during rupture. This process can be defined as the inversion of the two-phase nature of the medium and is a fundamental problem of explosion hydrodynamics, including a number of independent areas. One of them deals with the mechanism of the transformation of a foam structure into a drop structure. This is a sort of relaxation process, which calls for a detailed study. Getz and Kedrinskii [2] attempted to eliminate it in order to construct a model and analyze the dispersal of close packed drops and also proposed a model of instantaneous inversion of a cavitating liquid into a drop structure.

Another area involves numerical analysis of the parameters and structure of a cavitating liquid within the framework of the model of instantaneous relaxation of motion in the cavitation zone, making it possible to consider the dynamics of the zone up to high bulk concentrations [3]. As an example of this, we consider the problem of explosive projection of a liquid shell in the one-dimensional formulation.

Formulation of the Problem. A spherical explosive charge of initial radius r_0 lies at the center of a spherical liquid shell of radius r_1. After initiation of the charge at the center a detonation wave reaches the charge–shell contact boundary at time $t = 0$. The gas-kinetic flow formed at $t > 0$ is calculated for a wide range of $m = r_1/r_0$ ($m = 2$-10).

As the working medium we consider a real liquid, which is construed as a liquid with a natural content of microinhomogeneities of the type of microbubbles of free gas [4]. Their concentration is $\alpha_0 < 10^{-7}$. The effect of the compressibility of the gas component at such low values of α_0 is insignificant and so the propagation of shock waves in a real liquid is described well by the one-phase model. Taking this into account, we calculated the shock

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